HOLIDAY SPECIAL

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God, what a problem

What do you do with the world's hardest logic puzzle?
Make it even harder, says *Richard Webb*

NEED something to mull other than wine as you atrophy there in the armchair? Then set your grey cells humming with this puzzle.

"Three gods A, B, and C are called, in some order, True, False, and Random. True always speaks truly, False always speaks falsely, but whether Random speaks truly or falsely is a completely random matter. Your task is to determine the identities of A, B, and C by asking three yes-no questions; each question must be put to exactly one god. The gods understand English, but will answer all questions in their own language in which the words for 'yes' and 'no' are 'da' and 'ja', in some order. You do not know which word means which."

Welcome to the "Hardest Logic Puzzle Ever". If you should happen upon three questions that will unmask the gods, don't stop there. Your next task: make the puzzle even harder.

This is a parlour game played by logicians since the Hardest Logic Puzzle Ever was first so named – and solved – by US logician George Boolos shortly before his death in 1996. Find a solution, and you understand a little more about how to extract truth in a world where imperfect information abounds – and perhaps, by the by, about the nature of logic itself.
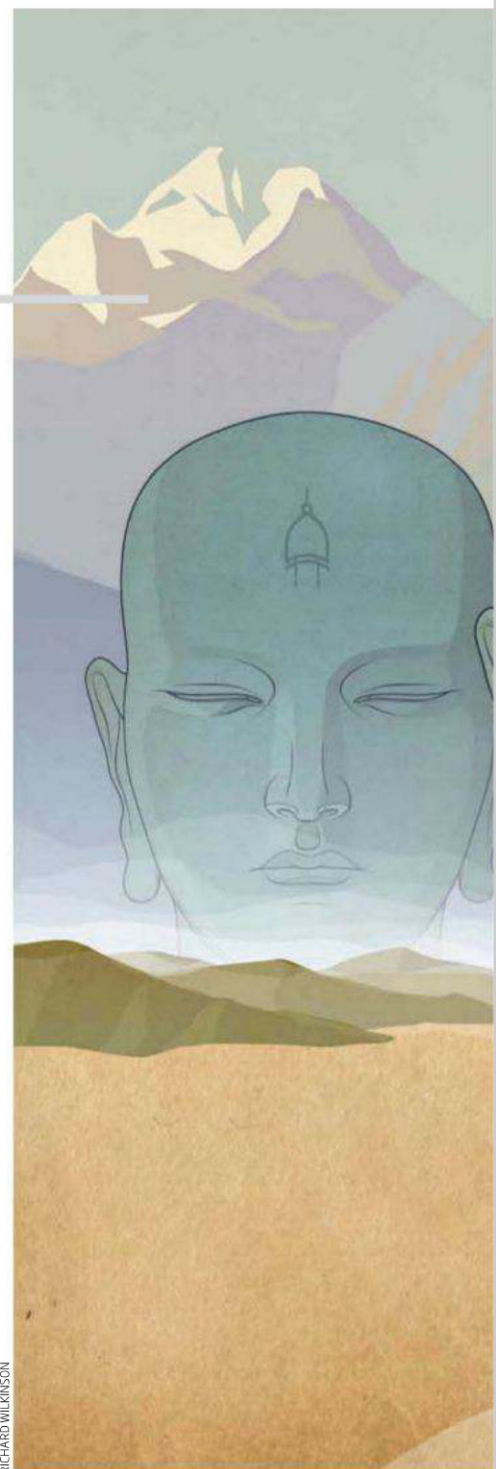
Boolos always had an individual take on the world. He once delivered a public lecture explaining Kurt Gödel's second incompleteness theorem, a seminal result in mathematical logic, entirely in words of one syllable, and was wont to pace the corridors of the Massachusetts Institute of Technology solving problems with invisible chalk on an invisible blackboard he always carried with him. In formulating the Hardest Logic Puzzle Ever, he was building on a series of mind-benders made popular by US mathematician Raymond Smullyan. In these puzzles, you are marooned on an island among knights, who always speak the truth, and knaves, who do nothing but lie. You generally have one question to extract some vital piece of information from them (see diagram, page 52).

Boolos's genius was to compact into one puzzle so many stumbling blocks that only a fiendishly complex series of questions can lead to the solution. "What makes it hard is the combination: liars and truth-tellers, language ignorance and finally a random element," says Brian Rabern, a philosopher at the University of Illinois at Urbana-Champaign. To reproduce Boolos's full answer, which he set out in *The Harvard Review of Philosophy*, would be to spoil the fun. For those needing a head start: his first question, addressed to god A, is, "Does 'da' mean 'yes' if and only if you are True if and only if god B is Random". Now get out those invisible chalkboards.

Boolos's intention in formulating the puzzle was not entirely frivolous. His solution stood or fell on extensive use of one of three classical axioms of logic attributed to Aristotle. Known as the law of excluded middle, it states that a logical proposition must be either true or false; there is no third way.
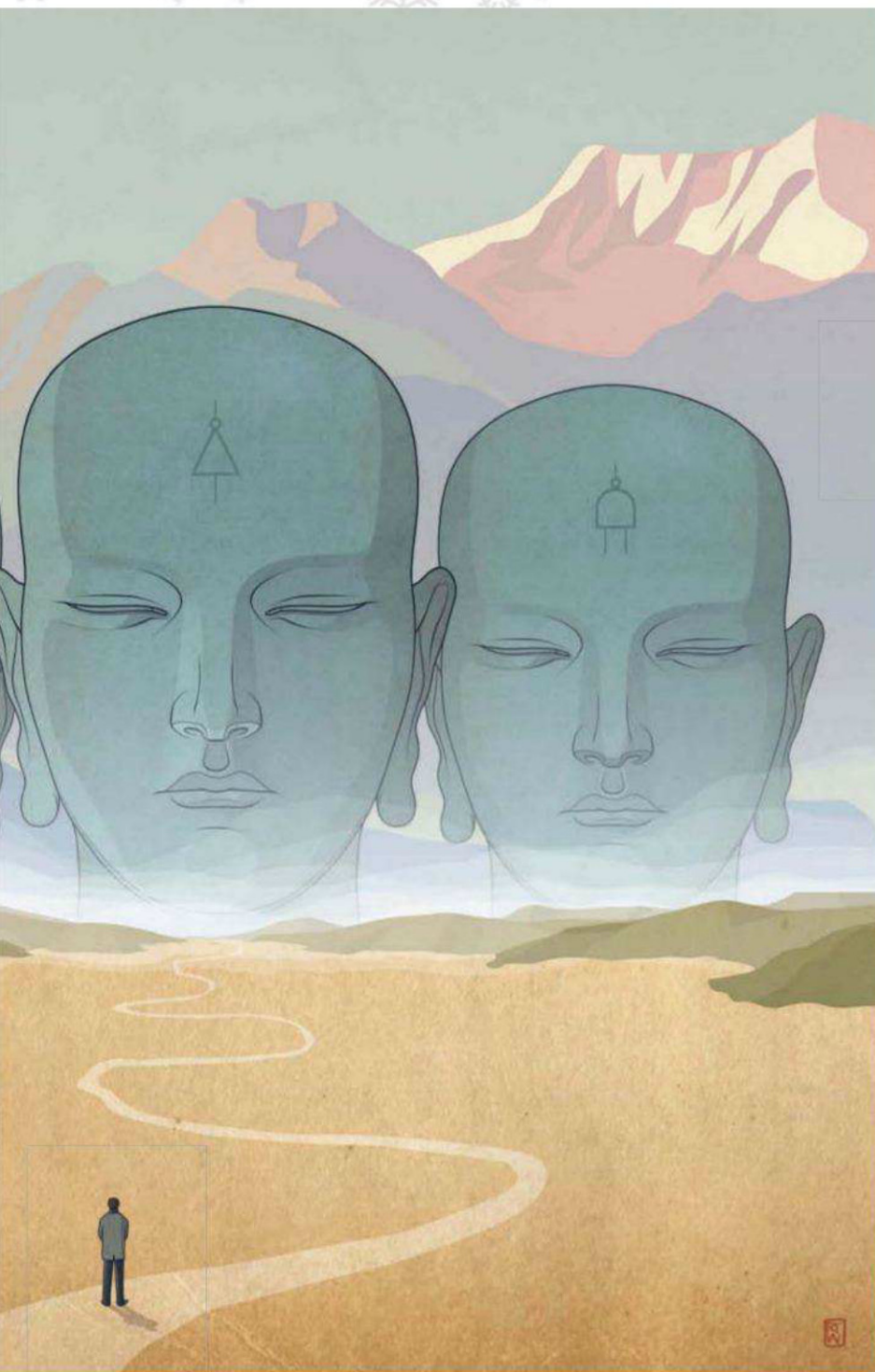
But is the law of the excluded middle itself true (and if not, is it false)? Consider, for example, the statement, "The present King of America has a beard". Is it necessarily false by virtue of there being no King of America, or does it lie in some grey zone between truth and falsehood? With his solution to the Hardest Puzzle, Boolos aimed to show how difficult it becomes to solve logical problems if one allowed such a middle way.

His solution was not to everyone's taste. As Tim Roberts of Central Queensland University in Bundaberg,

D

GETTY

The floodgates really opened in 2008, though, when Brian Rabern and his brother Landon discovered a more fundamental flaw in Boolos's original puzzle. It lay in his clarification of how Random generates his answers: like flipping a coin, Boolos specified, where heads makes him speak the truth and tails forces him to lie. In that case, said the Rabern brothers, just ask the question, "Are you going to answer this question with a lie?". True and False can only answer this question with the word meaning "no". If Random's coin shows heads, meanwhile, he must speak truly and also say the word for "no". Equally, if it shows tails, he must lie – again answering in the negative. So whoever you are speaking to, you now know how to say "no" in the gods' language.
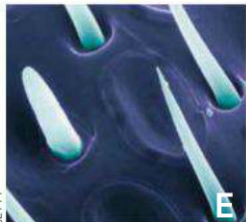
Head-exploders

This allows the problem to be solved in three surprisingly easy steps. That isn't all: similarly self-referential questions can also throw True and False into utter confusion. For example, ask them, "Are you going to answer 'ja' to this question?". If "ja" means "no" True cannot say the truth, and if it means "yes" False cannot say a lie, so one or other of them will be left lost for words. "We called them head-exploding questions," says Brian Rabern.

Such undefined statements are the bane of unwary computer programmers, producing a program that is paralysed by indecision. But the Rabern brothers showed how using these statements judiciously unmasked True and False quicker and helped to solve the puzzle in just two questions. Even when Random's behaviour was tweaked to make him answer truly randomly, the puzzle was easily solvable in three steps.

And so it went on. While the validity of head-exploding questions remains questioned (see "Explosive logic", page 52), in 2010 philosopher Gabriel Uzquiano of the University of Oxford embedded them in more complex logical structures to show that you could also solve the truly random version of the puzzle with just two questions – and then came up with a harder variant in which ➤

Australia, observed tartly in 2001, the obfuscating "if and only if" statements with which Boolos laced his solution were "the sort of thing that makes most laymen despair of logicians". Producing a solution that did away with them, Roberts concluded that the Hardest Puzzle was not so hard after all, and went on to suggest two more troublesome alternatives: make two gods Random, and the third either True or False; or one god Random and the other two indeterminately either True or False.

"George Boolos was wont to pace the corridors solving problems with invisible chalk on an invisible blackboard"

EXPLOSIVE LOGIC

Match this image to one on the grid on page 77, where you will find details of the competition

Random could randomly decide to say nothing at all. Later that year, Gregory Wheeler of Carnegie Mellon University in Pittsburgh, Pennsylvania, and his colleague Pedro Barahona responded with a solution to Uzquiano's problem in three questions. A still harder puzzle could be formulated, they suggested, by replacing Random with Devious, who lies when he can but if he gets confused acts like Random.

At the moment, they have their peers stumped with this version. "We have seen some papers come through, but nothing has quite got there," says Wheeler.

So what more is there to this, beyond logical one-upmanship? Quite a bit. "It is not just about logic, it is about information extraction, learning about nature when she is unwilling to give up her secrets,"

Solving George Boolos's "Hardest Logic Puzzle Ever" with "head-exploding" questions that have no true or false answer (see main story) puts the spotlight on the "law of non-contradiction". This axiom of classical logic states that no proposition may be both true and false. Graham Priest of the City University of New York is one logician who thinks it is at best a half-truth. He has spent the past three decades developing "paraconsistent" logical systems that admit the existence of dialetheia, or true contradictions. The initial motivation was to get around

the liar paradox – the 2500-year-old unsolved conundrum of what truth lies in the statement "this statement is false". "If you're using a paraconsistent logic, you can tolerate that sort of contradiction without it causing havoc elsewhere," says Priest. Some statements simply are both true and false.

Allowing some elasticity in our logic might help us to model the world better under certain circumstances: in quantum physics, for example, where things are not necessarily always one thing or the other, but sometimes a bit of both.

It is an approach that dismays purists. "Many theorists wouldn't like the idea of logic being held hostage to the empirical realm," says Brian Rabern, a philosopher at the University of Illinois at Urbana-Champaign.

That's a debate unlikely to produce a true or false answer soon. With the Hardest Puzzle, allowing a god to be a dialetheist unfazed by head-exploding questions shakes things up once again. "How do you solve the problem then?" asks Priest. "I've absolutely no idea, but it does ratchet things up a notch, which is nice."

says Wheeler. Brian Rabern agrees. "The god Random makes it a toy model of reasoning with imperfect information, which we must do all the time in normal life," he says. By clarifying how we do that most efficiently, the puzzle hones our logical arsenal – an understanding that could help us to program artificial intelligences to reason about the world.

That thought inspired Nikolay

Novozhilov, a hobby puzzler based in Singapore, to bind our hands even tighter. Earlier this year, he modified the puzzle's set-up so you are given no clues about the gods' language. This means we cannot ask questions such as, "If I asked you X, would you answer 'da'?". "The idea was to find out, if you eliminate all understanding, is it still solvable?" he says.

The answer is yes, provided whoever is being questioned has developed distinct ways to express basic logical concepts such as true and false. That result feeds into a long-running debate between linguists as to the minimum requirement to build a lexicon of a completely unknown language. Novozhilov playfully suggests one situation in which it might come in useful: that first encounter with the little green men. "I am sure that aliens will have the same understanding of logic whatever world they live in," he says. "Even if you don't have any information about how someone communicates, this shows there are features you can predict just from a logical understanding of what language is about."

Perhaps that is an unnecessarily unnerving suggestion with which to justify some fireside puzzle-solving. Wheeler suggests we need not look so far afield for a motivation. "There is something aesthetically lovely about a well-made puzzle." ∎

The world's hardest puzzle – easy version

George Boolos's "Hardest Logic Puzzle Ever" is an extension of simpler puzzles involving compulsive liars and inveterate truth-tellers
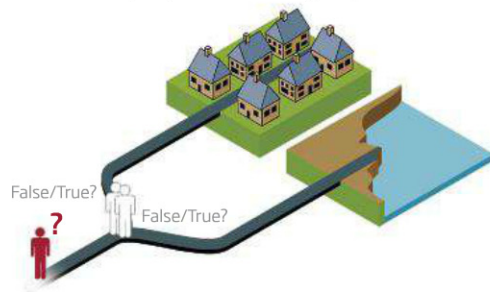
THE PROBLEM

A fork in a road is guarded by a liar (**False**) and a truth-teller (**True**) – you don't know which is which

What single question, demanding a yes or no answer, can you ask to find out which road leads to the village, and which over the cliff?

False/True?
False/True?
?

THE SOLUTION
Ask the question:

"If I ask the other person if the left path leads to the village, what would he say?"

	You ask True	You ask False
If the left path goes to the village	False would say no, so True says **NO**	True would say yes, so False says **NO**
If the right path goes to the village	False would say yes, so True says **YES**	True would say no, so False says **YES**

THE ANSWER
Regardless of who True and False are, the answer is: **NO = LEFT PATH** and **YES = RIGHT PATH**

Richard Webb is a features editor at *New Scientist*. For solutions to the "Hardest Puzzle" go to newscientist.com/dn22357